

## Lecture Outline

# Probability

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- Introduction
- Estimating probability
- Laws of probability
- Conditional probability
- Joint probability

## Introduction

- Probability quantifies the chances of an event happening
- Important to
  - Information theory
  - Statistics
  - Quantum physics
  - Everyday life

## Estimating Probability

- *a priori*
  - Estimated before samples are drawn
  - Know all possible outcomes and weightings
- Empirical
  - Determined by experiment / observation
  - Sample must be sufficient
- Subjective
  - Estimated without experiment

## Laws of Probability

- Stated as value between zero and one
  - never / always happen
- Probabilities sum to one
  - All possible events
- Mutually exclusive events
- Dependent / independent events

## Laws of Probability

- Additional law
  - Probability of one event *or* another event
  - Must be mutually exclusive
  - $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

## Laws of Probability

- Multiplication law
  - Probability of both one event *and* another event
  - Must not be mutually exclusive
  - Must be independent events
  - $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

## Laws of Probability

- General law of addition
  - Probability of one event *or* another event
  - Do *not* have to be mutually exclusive
  - $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

## Conditional Probability

- Outcome of one event is conditional on outcome of another
- Events are neither independent nor mutually exclusive
- *Information* of outcome of first event modifies probability of second event
- $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2 | E_1)$

## Joint Probability

- Probability two events occur
- Events can be either dependent or independent
- Must not be mutually exclusive
- For dependent events
  - $P(A \cap B) = P(B|A)P(A)$
- For independent events
  - $P(A \cap B) = P(A)P(B)$

## Summary

- Probability theory is widely used
- Heavily used in
  - Statistics
  - Information theory
- Laws describe how to calculate probabilities
- Occurrence of some events modify probability of other events