Chapter 6

Fuzzy Inference and Defuzzification

6.1 Introduction

Crisp Rules Revision
Fuzzy Sets revision
Fuzzy Inference
Fuzzy Rules
Fuzzy Composition
Defuzzification

6.2 Crisp Rules

-Consist of antecedents and consequents
-Each part of an antecedent is a logical expression
–e.g. A ¿ 0.5, light is on
-Consequent will be asserted if antecedent is true

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IF (Presentation is Dull) AND (Voice is Monotone)
   THEN Lecture is boring
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-Only one rule at a time allowed to fire
-A rule will either fire or not fire
-Have problems with uncertainty
-Have problems with representing concepts like small, large, thin, wide
-Sequential firing of rules also a problem
–order of firing

6.3 Fuzzy Sets

-Supersets of crisp sets
-Items can belong to varying degrees
–degrees of membership
–[0,1]
-Fuzzy sets defined two ways
–membership functions
—MF
–sets of ordered pairs
-Membership functions (MF)
-Mathematical functions
-Return the degree of membership in a fuzzy set
-Many different types in existence
–Gaussian
–Triangular
-Can also be described as sets of ordered pairs
-Pair Crisp,Fuzzy values
–A=(0,1.0),(1,1.0),(2,0.75),(3,0.5),(4,0.25),(5,0.0),(6,0.0),(7,0.0),(8,0.0),(9,0.0),(10,0.0)
-With enough pairs, can approximate any MF
-Fuzzification
-Process of finding the degree of membership of a value in a fuzzy set
-Can be done by
–MF
–Interpolating set of pairs

6.4 Fuzzy Rules

-Also have antecedents and consequents
-Both deal with partial truths
-Antecedents match fuzzy sets
-Consequents assign fuzzy sets
-Fuzzy rules can have weightings
–[0,1]
–importance of rule
–commonly set to 1
-Restaurant tipping example
-Antecedent variables are
–quality of service
–quality of food
-Consequent variables are
–Tip
-Service can be
–Poor
–good
–excellent
-Universe of discourse is 0-10
-Food can be

–rancid
–good
–delicious
-Universe of discourse is 0-10
-Tip can be
–cheap
–average
–generous
-Universe of discourse is 0-25
–% tip
-Rules for the tipping system

```
IF service is poor or food is rancid
  THEN tip is cheap
IF service is good
  THEN tip is average
IF service is excellent or food is delicious
THEN tip is generous
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6.5 Fuzzy Inference

-Infers fuzzy conclusions from fuzzy facts
-Matches facts against fuzzy antecedents
-Assigns fuzzy sets to outputs
-Three step process
–fuzzify the inputs (fuzzification)
–apply fuzzy logical operators across antecedents
–apply implication method
-Implication is really two different processes
–inference
–composition
-Inference is the matching of facts to antecedents
-Results in the truth value of each rule
–degree of support
–Alpha
-Assigns fuzzy sets to each output variable
-Fuzzy sets assigned to different degrees
-Determined by degree of support for rule
-Methods for assigning (inferring) sets
–min
–Product
-Min inference
-Cut output MF at degree of support
$\mu(v)' = min(z, \mu(v))$

Where:
$\mu(s)$ is the output MF
μ' is the inferred MF

v is the value being fuzzified
z is the degree of support

-Product inferencing
-Multiply output MF by degree of support
$\mu(v)' = z\mu(v)$

6.6 Tipping Example

-Assume
–service is poor
—score of 2
–food is delicious
—score of 8
-How do we perform fuzzy inference with these values?
-Firstly, fuzzify the input values
-Service fuzzifies to
–Poor 0.8
–Good 0.2
–Excellent 0.0
-Food fuzzifies to
–Rancid 0.0
–Good 0.4
–Delicious 0.6
-Now, calculate the degree of support for each rule
-Rule 1:
-IF service is poor or food is rancid
–poor = 0.8
–rancid = 0.0
–max(0.8, 0.0) = 0.8
–Degree of support = 0.8
-Now, calculate the degree of support for each rule
-Rule 1:
–IF service is poor or food is rancid
–poor = 0.8
–rancid = 0.0
–max(0.8, 0.0) = 0.8
–Degree of support = 0.8
-Rule 3
–IF service is excellent or food is delicious
–excellent = 0.0
–delicious = 0.6
–max(0.0, 0.6) = 0.6
–Degree of support = 0.6
-Apply implication method
-Builds an inferred fuzzy set
-Find the min value for each output MF
-Cut output MF at this value

6.6.1 Min Inference

-Cut at 0.8
–fig
-Corresponding fuzzy set
–MF = (0,0),(1,0.2),(2,0.4),(3,0.6),(4,0.8),(5,0.8),(6,0.8),(7,0.6),(8,0.4),(9,0.2),(10,0), (25,0)
-Degree of support of 0.4
–fig
-Corresponding set
–MF = (0,0),(1,0.2),(2,0.4),(3,0.4),(4,0.4),(5,0.4),(6,0.4),(7,0.4),(8,0.4),(9,0.2),(10,0), (25,0)

6.6.2 Product inference

-How are things different if we use product inferencing?
–fig
-Corresponding set
–MF = (0,0),(1,0.16),(2,0.32),(3,0.48),(4,0.64),(5,0.8),(6,0.64),(7,0.48),(8,0.16),(9,0.16),(10,0), (25,0)
-Degree of support of 0.4
–fig
-Corresponding set
–MF = (0,0),(1,0.08),(2,0.16),(3,0.24),(4,0.32),(5,0.4),(6,0.32),(7,0.24),(8,0.16),(9,0.08),(10,0), (25,0)

6.7 Fuzzy Composition

-Aggregates the inferred MF into one
-Two methods of doing this
–Max
–Sum
-MAX takes the max fuzzy value for each value of v
–equivalent to taking the fuzzy values for the highest activated rule for each output fuzzy set
-SUM sums all fuzzy values for each value of v
–can lead to truth values ¿ 1
–may need to be normalised to [0,1]
—implications for defuzzification
-Assume
–3 MF attached to the output
—A, B and C
–Each MF has been asserted by two different rules
—6 rules activated (degrees of support) ¿ 0
–Degrees of support
—0.8, 0.4, 0.6, 0.5, 0.7, 0.3
–Prod inference used

-for set A
–table
-for set B
–table
-for set C
–table

6.7.1 MAX Composition

MAX composition
Take the max of each column
–table
–fig
-Sum composition
–sum each column
–table
–fig

6.8 Defuzzification

-Converts inferred MF into crisp numbers
-Many different types in existence
-Two common ones
–Centre of Gravity
–Mean of Maxima

6.8.1 COG Defuzzification

-Centre of Gravity
–CoG
$y = \frac{\sum_i^K \mu(v_i)v_i}{\sum_i^K \mu(vi)}$
-Where:
–y is the crisp value
–K is the number of items in the fuzzy set
-Applying this to the first composite set
–table
$\sum_i^K \mu(v_i)v_i = 121.68$
$\sum_i^K \mu(v_i) = 10.1$
$\frac{121.68}{10.1} = 12.05$
–fig

6.8.2 MoM Defuzzification

-Mean of Maxima
–MoM

-Finds the mean of the crisp values that correspond to the maximum fuzzy values
-If there is one maximum fuzzy value, the corresponding crisp value will be taken from the fuzzy set
-Applying this to the first composite set
-Maximum fuzzy value is 0.8
-Corresponding crisp value is 4
-This is the value returned by MoM
-What about sets with ¿ 1 maximum?
-Apply this to the third composite set
–table
-Maximum fuzzy value if 0.8
-Corresponding crisp values are
–4, 5 and 6
$y = \frac{4+5+6}{3} = 5$
–fig

6.9 Summary

-Fuzzy rules match fuzzy antecedents to fuzzy consequents
-Degree to which antecedents are true determine the degree of support
-Fuzzy logic functions are used to determine this
-Fuzzy inference involves calculating an output fuzzy set
-Different inference process produces different inferred MF
-Two inferences processes are
–max-min
–Max-prod
-Two common composition methods
–MAX
–SUM
-Inference methods described by combining inference & composition methods
–max-min (or min-max)
–max-prod
-Defuzzification converts a composed MF to a single crisp value
-Different defuzzification methods produce different crisp values
–sometimes wildly different
-Two different defuzzification methods
–Centre of Gravity
—CoG
–Mean of Maxima
–MoM