

Chapter 5

Fuzzy Sets and Fuzzification

5.1 Introduction

- Crisp sets
- Fuzzy sets
- Fuzzy membership functions
- Fuzzification
- Fuzzy logic

5.2 Crisp Sets

- Everything is either true or false
- No uncertainty is allowed
- An item either is
 - entirely within a set, or
 - entirely not in a set
- The Law of the Excluded Middle
- X must be either in set A or in set not-A
- no middle ground is allowed
- Opposite sets (A and not-A) must between them contain everything
- Venn diagrams

5.3 Fuzzy Sets

- Items can belong to a fuzzy set to different degrees
 - degrees of membership
 - Completely within a set is a membership degree of 1
 - Completely outside a set is a membership degree of 0
 - Degrees of membership must sum to 1
 - An item can be both A and not-A to different degrees
 - e.g. A to a degree of 0.8, not-A 0.2
 - Degrees of membership are expressed with membership functions

-Range of values a variable can take is called the universe of discourse

5.4 Membership Functions

- A membership function describes the degree of membership of a value in a fuzzy set
- Referred to as MF
- Also $\mu(x)$
- where x is the value being fuzzified
- here are many different types of MF
- Which one to use depends on the problem

5.4.1 Singleton MF

5.4.2 Rectangular MF

5.4.3 Triangular MF

5.4.4 Gaussian MF

5.4.5 Other Representations of MF

5.5 Fuzzification

- The process of determining the degree to which a value belongs in a fuzzy set
- The value returned by a fuzzy MF
- Most variables in a fuzzy system have multiple MF attached to them
- Fuzzifying that variable involves passing the crisp value through each MF attached to that value

5.6 Fuzzy Logic

- Same operations and function as in crisp logic
- Must deal with degrees of truth rather than absolute truths
- Fuzzy logic is a superset of crisp (Boolean) logic
- A-ND, OR, NOT
- Crisp logical functions
- AND true if both parameters are true
- OR true if either parameter is true
- NOT reverses truth of argument
- AND function - crisp version
- AND function - fuzzy version
- take the minimum of the two arguments
- OR function - crisp version
- OR function - fuzzy version
- take the maximum of the two arguments
- NOT function - crisp version

- NOT function - fuzzy version
- subtract the truth value from one
- Output of fuzzy logical functions are the same as crisp functions
- just calculated differently
- handle degrees of truth, rather than absolute truths
- The basis of fuzzy rule based systems

5.7 Summary

- Fuzzy logic deals with uncertainty
- Allows degrees of truth
- Allows partial membership in sets
- Fuzzy membership functions describe degrees of membership in fuzzy sets
- Many different types of MF exist
- Fuzzification = determining degree of membership
- uses fuzzy MF to do so
- Fuzzy logic extends Boolean operators to handle partial truths
- the basis of fuzzy rules

