Chapter 3

Non-Linear Transformations, Fourier and Wavelet Transformations

3.1 Introduction

-Linear vs Non-linear transformations
-Non-linear transformations
-Image transformations
-Fourier and Wavelet transforms

3.2 Linear vs Non-Linear Transformations

-Linear transformations use a linear function
-Non-linear functions use a non-linear function
-Linear transformations maintain the distribution of the data
—Can stretch the distribution
–Multiplicative transformation
–Can shift the distribution
—Additive transformation
-Non-linear transformations alter the distribution
-Can make the distribution normal
–Why do this?
—Remove outliers
-Can make the distribution uniform
–Why do this?
—Image processing
-Only applicable to ratio scale or above
–Require true zero points
—Log of negatives?
-Examples of transforms
–Log

–Exponential
—Inverse of log
–Binomial
–Tanh

3.3 Image Transformations

-Many image processing transformations are non-linear
–Why?
-Examples
–Convolution
–Sobel filters
–Median filters
-Convolution
–General technique
–Uses a small matrix
—Kernel
–Kernel is slid over the image
—Moves one pixel at a time
–Values in the kernel are used to transform values in image
–Basis of many image processing techniques
-Sobel filter
–Based on a convolution
–Edge detector
–Edges have high contrast
–Measures the gradient between adjacent groups of pixels
–Uses specific kernel values
-Median filters
–Family of filters
–Noise reduction
—Impulse noise
–Examine groups of pixels
–Remove spikes in intensity
–Different methods used
—Not all non-linear

3.4 Signals and Waves

3.4.1 Signals

-What is a signal?
-Change in a variable over time
-Many things can be treated as signals
–Sound
–Light
–Images
–DNA

-Signal processing is an important field in computing

3.4.2 Properties of Waves

-Displacement in a medium
-Waves are signals
-Waves can interfere with one another
–Constructive
–Destructive
–Beats
-Complex waves can be created
–Use a large (possibly infinite) number of component waves
-Visualisation of waves
–Time domain
–Frequency domain
-Time domain is traditional
–Amplitude vs time
-Frequency domain is more useful for analysis

3.5 Fourier Transformations

-Convert a signal into the frequency domain
-Separates signal in components
-Useful for many things
–Analysis of the signal
–Noise suppression
–Signal modeling
—Speech recognition
-Discrete Fourier Transforms
–DFT
-Used for discretised signals
–Ergo, digital signals
-Fast Fourier Transforms
–FFT
-Efficient implementation of the DFT
–Does the same thing quicker

3.6 Wavelet Transformations

-What is a wavelet?
-Two main properties
–Limited duration
–Average value of zero
-A wavelet fades in and fades out
-Wavelet analysis breaks a signal into its component wavelets

-Wavelet coefficients can be recombined to form the original signal
-Wavelets have many applications
–Noise filtering
–Compression
-Wavelets are used when FFT don’t work

3.7 Summary

-Non-linear transforms alter the distribution of data
-Often used to transform images
-Many image transformations are based on convolution
-Fourier and wavelet transforms are other non-linear transformations
-Fourier transforms break signals into component sinusoids
-Used in many signal processing applications
-DFT applied to DSP
-FFT is an implementation of DFT
-Wavelet transforms break signal into component wavelets
-Used in signal compression among others